

# A few-shot knowledge reasoning method based on three-way partial order structure and prompt learning<sup>☆</sup>

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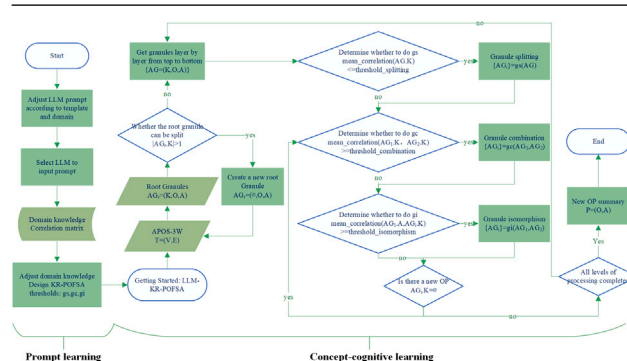
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## HIGHLIGHTS

- Combination of prompt learning and concept-cognitive learning.
- LLM-enhanced knowledge reasoning based on granular computing.
- Few-shot knowledge reasoning from perspective of concept learning.

## GRAPHICAL ABSTRACT



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## ABSTRACT

As an emerging topic, few-shot knowledge reasoning is of great significance to the advancement of artificial intelligence. KR-POFSA is an approach that employs a three-way partial order structure for knowledge representation, combined with granular computing to achieve few-shot knowledge reasoning. However, it faces a limitation: when using it for knowledge reasoning, if the number of attributes is huge or there are a lot of cross-relationships between the attributes, then it may generate lots of new potential object patterns, most of which are useless. To solve this problem, this paper devises an innovative approach to improve KR-POFSA, which uses LLM to downstream few-shot knowledge reasoning tasks through appropriate prompt design. Specifically, we design a prompt template that guides LLM to output domain knowledge, such as a correlation matrix, and uses thresholds to limit the generation of invalid patterns. Through three experiments—with 8 objects and 9 attributes, 20 objects and 11 attributes, and 23 objects and 12 attributes, respectively—we demonstrate that our method can not only reduce the discovery of invalid attribute granules and object patterns in granular computing by 30 %–50 %, but also may offer practitioners insights into which attributes to prioritize, minimizing empiricism.

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## 1. Introduction

Few shot learning (FSL) [29], as a learning model for small sample size problems, is a vital and difficult problem in artificial intelligence (AI). The huge amount of data produced in the era of big data has made AI technology develop rapidly, which has led to the current situation where most models in the domain of AI only rely on a large number of samples for sufficient learning. However, in certain specific fields such as healthcare, facial recognition, and anomaly detection, it is quite not easy to acquire a large number of samples [16]. To solve this problem, scholars have started to consider whether knowledge learned from small samples is helpful and how to learn knowledge from small samples. This conjecture has aroused widespread research interest among scholars, thus the idea of few-shot learning has emerged by degrees [19].

Knowledge reasoning, the process of deducing new insights from existing information, is the cornerstone of AI. Humans can infer new knowledge from minimal examples, while replicating this capability in AI systems is far from trivial. Most existing knowledge reasoning methods find it difficult to extract high-order information from small sample data, which limits their inference performance greatly. The method of few-shot learning provides a guideline to solve knowledge reasoning problems on small sample data. Therefore, few-shot knowledge reasoning, composed of knowledge reasoning and few-shot learning, is worth further study.

At present, some studies on few-shot knowledge reasoning have achieved preliminary results, such as few-shot inductive link prediction via meta learning [50], semantic matching network for few-shot knowledge graph completion [24], multi-hop knowledge graph reasoning in few-shot scenarios [49], few-shot multi-hop reasoning via reinforcement learning and path search strategy over temporal knowledge graphs [1]. However, most of the above achievements focus on few-shot reasoning from the perspective of vector representation, whose interpretability is relatively low and cannot be applied in fields that require high interpretability, including syndrome differentiation and treatment in traditional Chinese medicine (TCM) [37]. Compared with vector-based reasoning [21], symbol-based reasoning [46] has relatively high interpretability. Hence, it is a vital task to study few-shot from the perspective of symbol representation, such as concept-based few-shot knowledge reasoning.

Concepts, as the cornerstone of human cognition, play a crucial role in the process of knowledge representation and reasoning. Concept-cognitive learning (CCL) [7], as a novel AI field of interdisciplinary research that has emerged in recent years, involves multiple methods such as rough set, fuzzy set [9,30], granular computing (GrC) [3], formal concept analysis (FCA) [32], three-way decision(3WD) [27,42], two way learning [34], cognitive science [38], attention mechanism [33], attribute topology [48], incremental learning [25,39], and representation learning [35], and has achieved fruitful results in fields such as knowledge clustering [3], concept clustering [23], multi-label classification [32], information fusion [30], knowledge discovery [9].

Partial order formal structure analysis (POFSA)[35,38–41], proposed by the team of Prof. Hong, is an innovative CCL model and is closely related to various knowledge processing approaches such as knowledge embedding [35], knowledge distance [40], and knowledge visualization [38]. As a symbolic representation model, it has been widely used in knowledge processing of various fields such as linguistic knowledge discovery [45], word sense disambiguation [44], and TCM knowledge mining [36].

There has explored the mechanism of few-shot knowledge reasoning from the perspective of symbol representation using POFSA, and proposed a few-shot knowledge reasoning method: KR-POFSA, which leveraged a mathematically grounded and interpretable framework for reasoning, offering significant potential for few-shot learning scenarios. Nevertheless, when executing KR-POFSA, if there are a large number of attributes and the cross relationships between attributes are complex,

many new possible patterns will arise, many of which are meaningless. Due to the significant manpower, resources, and time required to validate the new possible patterns, the practical value of KR-POFSA is greatly limited. Therefore, it is necessary to seek measures to improve KR-POFSA in order to reduce unnecessary new pattern generation and ultimately enhance the reasoning performance of KR-POFSA [41].

Recently, prompt learning [53], as an emerging natural language processing technique, can guide pre-trained language models (PLMs) [28] such as large language models (LLMs) to learn in few-shot or even zero-shot scenarios by designing specific, simple yet efficient prompts. Through the scholarly verification of various fields, appropriate prompt design can stimulate the PLM's powerful semantic understanding and generation abilities, enhance the PLM's knowledge transfer and cross domain adaptability, and help PLMs adapt to various downstream tasks, such as classification, reasoning, and more. Therefore, prompt learning gradually receives widespread attention from researchers [2,12].

### 1.1. Research objectives

Inspired by prompt learning, this article will explore an LLM-enhanced few-shot knowledge reasoning method based on POFSA. The novel method will attempt to adapt the LLM to downstream few-shot knowledge reasoning tasks through appropriate prompt design, so as to enhance the reasoning performance of KR-POFSA.

### 1.2. Key contributions

The key contributions can be summarized as follows:

- (1) Combination of prompt learning and concept-cognitive learning.
- (2) LLM-enhanced knowledge reasoning based on granular computing.
- (3) Few-shot knowledge reasoning from perspective of concept learning.

### 1.3. Article structure

This article adopts a hierarchical framework comprising six analytical components. Section 1 provides an introduction to our study's contextual foundations, research objectives and key contributions, Section 2 describes some fundamental theoretical concepts and essential prerequisites, Section 3 presents the few-shot knowledge reasoning method based on POFSA and prompt learning, Section 4 details three experiments carried out concretely, Section 5 discusses the proposed methods and conducted experiments, finally, Section 6 concludes to study.

## 2. Preliminaries

This section introduces some fundamental concepts employed in this article.

### 2.1. Prompt

Prompt is an AI-based technology that directs the output of PLM through clear and explicit instructions. Prompt learning, originating from in-context learning (ICL) [4], can enhance the efficacy of PLMs without modifying the core parameters, bridge the gap between PLMs and downstream tasks, and unleash the potential of PLMs. More and more scholars are conducting research on prompts, not only in the field of natural language processing (NLP) but also in other fields such as computer vision (CV) [18].

In the past, if you wanted to apply a PLM to a different task, this usually required retraining the model or fine-tuning. But now we often see seamless integration of PLMs into downstream tasks by prompts. As shown in the Fig. 1, prompts can be specific regions or features within images, or descriptive text that serves as guiding signals for vision models, helping them to generate or analyze visual content in CV, while prompts in the field of NLP are usually natural language instructions that guide model output.

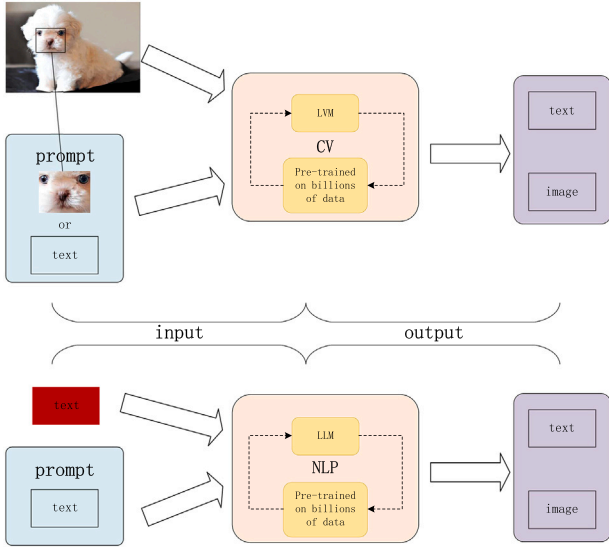


Fig. 1. Prompt in CV &amp; NLP.

In prompt engineering, a well-designed prompt integrates key elements such as role, task, example, and input problem, aligning the model's response with specific requirements. From the perspective of structured thinking prompts, prompt learning can be roughly divided into two main categories: the first type is to add additional text as prompts at the input of the PLM, while the second type is to introduce additional frameworks or methods based on the basic prompt text to more effectively guide the PLM in completing tasks.

Due to the simplicity of our requirements, in this article, we use the first type: the prompt template we propose is also a natural language to guide PLM to output domain knowledge.

## 2.2. Domain knowledge

Domain knowledge [15], encompasses the understanding and expertise within a specific field, manifesting in various forms such as linear and non-linear equations, constraints of equality and inequality, and logical formulas. The incorporation of domain-knowledge has been one of the 3 Grand Challenges in developing AI systems [52].

Domain knowledge is usually provided by experts in the field, while non-experts can sometimes offer simple insights or "rules of thumb". Given the multidisciplinary nature of AI data, it poses a challenge for AI scholars to acquire comprehensive domain knowledge. LLMs, as the typical representatives of PLMs, characterized by their extensive parameters and intricate computational architecture, potentially harbors domain knowledge due to their vast scale of data and parameters. For LLMs, domain knowledge is incorporated into deep neural networks by encoding it as a set of propositional rules. Taking Knowledge-Based Artificial Neural Network (KBANN) as an example, domain knowledge is represented as hierarchically structured propositional rules that directly determine the fixed topological structure of a neural network [13,43].

In this article, domain knowledge is primarily represented in the form of a two-dimensional matrix. We will explore the generation of domain knowledge by guiding LLM through the design of structured thinking prompts and integrating the generated domain knowledge into KR-POFSA.

## 2.3. CCL

For understanding the content of the following text easily, it is necessary to first understand the relevant concepts of CCL [5,8,31].

Table 1

A formal context: Live in Water.

|   | a | b | c | d | e | f | g | h | i |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 |   |   |   |   | 1 |   |   |
| 2 | 1 | 1 |   |   |   |   | 1 | 1 |   |
| 3 | 1 | 1 | 1 |   |   |   | 1 | 1 |   |
| 4 | 1 |   | 1 |   |   |   | 1 | 1 | 1 |
| 5 | 1 | 1 |   | 1 |   | 1 |   |   |   |
| 6 | 1 | 1 | 1 | 1 |   | 1 |   |   |   |
| 7 | 1 |   | 1 | 1 | 1 |   |   |   |   |
| 8 | 1 |   | 1 | 1 |   | 1 |   |   |   |

**Definition 2.1.** *Formal context.* For a given triad  $K = \{U, M, I\}$ , it is a binary formal context, or you can call it Boolean formal context, if  $U$  and  $M$  are sets of objects and attributes, when  $I$  is binary relationship over  $U \times M$ , which can be abbreviated as formal context, and often exists in the form of a two-dimensional table like Table 1.

**Definition 2.2.** *Cognitive operator.* Given a formal context  $K = \{U, M, I\}$ , for object set  $U$  and attribute set  $M$ , according to the mapping relations between their respective power sets  $2^U$  and  $2^M$ , two cognitive operators can be defined:  $f : 2^U \rightarrow 2^M$  and  $g : 2^M \rightarrow 2^U$ . For arbitrary  $A \subseteq U$  and  $B \subseteq M$ , the two cognitive operators are described as:

$$\begin{cases} f(A) = \{m \in M | \forall u \in A, (u, m) \in I\} \\ g(B) = \{u \in U | \forall m \in B, (u, m) \in I\} \end{cases} \quad (2.1)$$

**Definition 2.3.** *Cognitive concept.* Given a formal context  $K = \{U, M, I\}$ , for arbitrary  $A \subseteq U$  and  $B \subseteq M$ ,  $(A, f(A))$  and  $(g(B), B)$  can be called incomplete cognitive concepts [38], and if they satisfy:  $f(A) = B$  and  $g(B) = A$ , then  $(A, f(A)) = (g(B), B) = (A, B)$  can be called complete cognitive concept:  $A$  is concept extent, while  $B$  is concept intent.

## 2.4. POFSA

For the given two concepts  $(A_1, B_1)$  and  $(A_2, B_2)$ , the partial order relation ( $\leq$ ) between them can be described as:  $(A_1, B_1) \leq (A_2, B_2) \Leftrightarrow A_1 \subseteq A_2 \Leftrightarrow B_2 \subseteq B_1$ . According to the theory of POFSA, based on the partial order relations between various concepts, all the attributes in the formal context of Table 1 can be organized into one kind of granular structure as shown in Fig. 2: attribute partial order structure (APOS) [38].

**Definition 2.4.** *Attribute granule (AG).* For a given APOS, each node can be expressed with a triad:  $AG = (K, O, A)$ , among which  $K$  is the key attribute, while  $(O, A)$  represents the corresponding concept:  $O$  and  $A$  are the extent and intent, respectively.

Take Fig. 2 as example, the second top layer of APOS and APOS-3W has two and three AGs respectively: in APOS, they are  $AG_{11} = (b, 12356, ab)$ ,  $AG_{12} = (c, 478, ac)$  in APOS; while in APOS-3W, they are  $AG_{11} = (b, 125, ab)$ ,  $AG_{12} = (c, 478, ac)$ ,  $AG_{13} = (bc, 36, abc)$ .

The main difference between APOS and APOS-3W is the number of key attributes in each AG: there is only one in APOS, whereas there may be multiple ones in APOS-3W.

**Definition 2.5.** *Object pattern (OP).* For a given APOS, each branch can be expressed with a two-tuple:  $OP = (O, A)$ . The  $O$  and  $A$  have the same meaning as AG, which can simply understand it as an object set and an attribute set. Sometimes you can add the count of objects ( $|O|$ ) and express  $OP$  as  $OP = (c, O, A)$ .

Take Fig. 2 as example, for APOS, from left to right, the  $OP$  of each branch can be described as:  $OP_1 = (6, abcd f)$ ,  $OP_2 = (3, abcd gh)$ ,  $OP_3 = (5, abcd f)$ ,  $OP_4 = (2, abcd gh)$ ,  $OP_5 = (1, abcd)$ ,  $OP_6 = (7, acde)$ ,  $OP_7 = (8, acd f)$ ,  $OP_8 = (4, acd ghi)$ ; while for APOS-3W, ignoring the order of  $OP$ , they are the same as those of APOS.

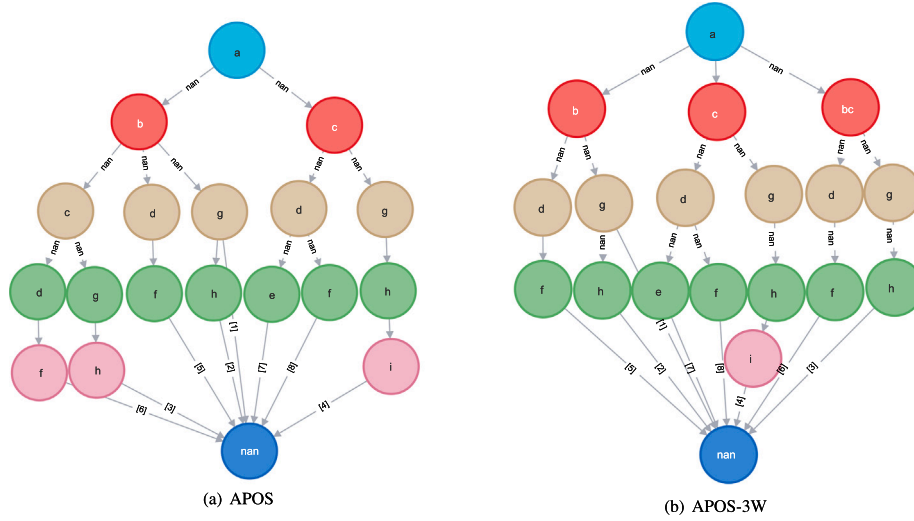


Fig. 2. Examples of APOSs in POFSa.

**Definition 2.6.** *Attribute partial order structure (APOS).* It is a hierarchical tree structure, its nodes correspond to  $AG$ s and branches represent  $OP$ s.

**Definition 2.7.** *Three-way attribute partial order structure (APOS-3W).* It is an APOS enhanced via GrC-based multi-granularity and 3WD-based triadic partitioning [38].

Fig. 2 APOS (left) and APOS-3W (right) are generated from the formal context ‘Live in water’ (Table 1) using the algorithm [38]. In order to better represent the  $OP$  represented by APOS and APOS-W branches, we can add an empty node on the last layer like Fig. 2 or add at the end of each branch like Figs. 7–10. These nodes are not  $AG$ s, they just make it easier for us to see the extent of the  $OP$  directly.

## 2.5. KR-POFSa

For realizing few-shot knowledge reasoning with APOS (KR-POFSa), there are some knowledge reasoning operators to be defined [41]:

**Definition 2.8.** *Granule splitting ( $gs$ ).* For a given attribute granule  $AG = (K, O, A)$  in APOS, if it satisfies  $|AG.K| > 1$ , then it can split out to generate new granules through the following Eq. (2.2), where  $2^K$  is the power set of  $K$ .

$$gs(AG) = \{AG_i | AG_i.K \subseteq 2^K, AG_i.A = AG_i.K \cap AG.A, AG_i.O = AG.O \cup ?, i < 2^{|K|} - 1\} \quad (2.2)$$

**Definition 2.9.** *Granule combination ( $gc$ ).* For two given attribute granules  $AG_1$  and  $AG_2$  in APOS, if they meet: (a) positioned within the same layer; (b) each specially appointed to an  $OP$ ; (c) share a same parent granule, then they can be combined and then generate a new granule by the following Eq. (2.3).

$$gc(AG_1, AG_2) = \{AG_i | AG_i.K = AG_1.K \cup AG_2.K, AG_i.A = AG_1.A \cup AG_2.A, AG_i.O = ?, i = 1\} \quad (2.3)$$

**Definition 2.10.** *Granule isomorphism ( $gi$ ).* For two given attribute granules  $AG_1$  and  $AG_2$  in APOS, if they meet: (a) positioned within the same layer; (b)  $AG_1.K \cap AG_2.K \neq \emptyset$ , then they can expand their own sub-granules according to the core attributes of the other sub-granules via the following Eq. (2.4), where  $CG_1$  and  $CG_2$  are the child granules of  $AG_1$  and  $AG_2$ .

$$\begin{cases} gi(AG_1, AG_2) = \{AG_i | AG_i.K \subseteq ((CG_2.K) - \{CG_1.K\}), AG_i.A = AG_i.K \cup AG_1.A, AG_i.O = ?, i \leq |CG_2| - 1\} \\ gi(AG_2, AG_1) = \{AG_i | AG_i.K \subseteq ((CG_1.K) - \{CG_2.K\}), AG_i.A = AG_i.K \cup AG_2.A, AG_i.O = ?, i \leq |CG_1| - 1\} \end{cases} \quad (2.4)$$

**Definition 2.11.** For  $AG$  generated by  $gs$ ,  $gc$ ,  $gi$ , if  $AG.K = \emptyset$ , then  $AG$  is a possible  $OP$  and can be described as:  $OP = (AG.O, AG.A)$ .  $OP$  is valid if it meets one of the following conditions: (a)  $\exists u \in U \subseteq K = \{U, M, I\}$ , s.t.  $f(u) = OP.A$ , in this case, we can know that  $u \in OP.O$ ; (b)  $OP.A \subseteq \mathcal{D}_{external}$ , where  $\mathcal{D}_{external}$  is external authoritative domain knowledge base. It is invalid when it meets the following conditions at the same time: (a)  $\forall u \in U \subseteq K = \{U, M, I\}$ , s.t.  $f(u) \neq OP.A$ ; (b)  $\exists a_i, a_j \in OP.A$ , s.t.  $a_i \perp a_j \vee OP.A \not\subseteq \mathcal{D}_{external}$ .

For a given formal context, to realize few-shot knowledge reasoning, firstly, an APOS-3W should be constructed according to the generation algorithm [38]; then based on the APOS-3W, KR-POFSa can be realized for few-shot knowledge reasoning as shown in Fig. 3. Here we give the flowchart of KR-POFSa. The process starts with obtaining the root node  $AG_r = (K, O, A)$  and first determines whether the root node is divisible ( $|AG_r.K| > 1$ ). If it is divisible, a new root node is created; otherwise, the  $AG$ s of each layer are obtained from top to bottom. The  $AG$ s are then processed through three operations:  $gs$ ,  $gc$ , and  $gi$ . If a new  $OP$  is generated after processing, it is added to  $P = (O, A)$  and  $gc$  and  $gi$  are repeated. Otherwise, the next layer is processed until all layers are processed. Finally it will return a set of new  $OP$ s. More detailed information can be found in the reference [41].

## 3. Methods

When dealing with numerous attributes and their cross-relations, KR-POFSa tends to generate a large number of new  $OP$ s, many of which are invalid. Therefore, in this section we will explore how to use prompts to generate domain knowledge, and then constrain the number of new  $OP$ s that KR-POFSa generates, so as to achieve the goal of reducing the generation of invalid  $OP$ s while preserving valid  $OP$ s as much as possible.

### 3.1. Prompt for domain knowledge

As mentioned above, prompts generally include role, task, example, and problem. So we will take this to design prompt templates and adjust them for specific fields when using them. For KR-POFSa, we need



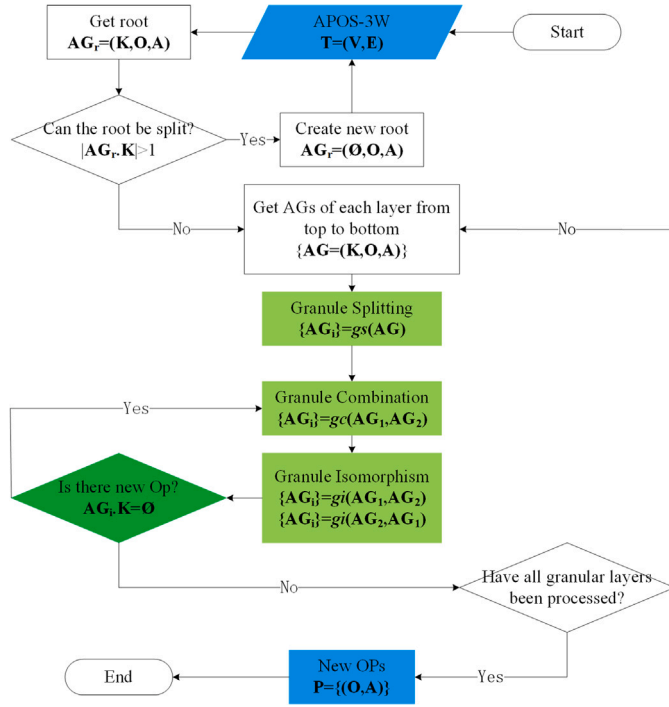


Fig. 3. The flowchart of KR-POFSA.

to obtain a two-dimensional matrix of domain knowledge, role is an expert in the relevant field, task is responsible for the expected output: a two-dimensional matrix of domain knowledge, which is the correlation between attributes (between 0 and 1), example is general, and problem is input: some attributes. Obviously, role, task, and problem are what we need to adjust. Fig. 4 shows us how to use prompt templates to generate domain knowledge.

Table 2 provides the prompt template and an example we use. In this example our goal is to obtain the domain knowledge of the Live in water dataset (as shown in Table 1), that is, the correlation of the attributes in this dataset. The dataset shown in Table 1 is a two-dimensional relationship between organisms and their attributes: 0 indicates that the organism does not possess this attribute, and 1 indicates that it does. Thus, it is clear that for our purposes, the LLM is going to play the role of a biologist and assess the likelihood of pairs of attributes being on the same organism, and finally we need to provide it with those attributes.

The domain knowledge will be generated as a two-dimensional matrix by LLM, it can be called correlation matrix. We can use it to reduce the number of invalid  $OPs$  that satisfy  $\exists a_i, a_j \in OP.A, s.t. a_i \perp a_j$  while trying to preserve other  $OPs$ .

### 3.2. Thresholds in KR-POFSA

After obtaining domain knowledge, we can make slight adjustments to it. At the same time, we can design thresholds based on it, including thresholds for  $gs$ ,  $gc$ , and  $gi$ . In this section, we will optimize the three knowledge reasoning operations mentioned in KR-POFSA according to these thresholds.

First, for the  $gs$  operation, the core of this operation is to split a granule into multiple ones according to the key attributes of the granule. In order to constrain the number of results produced by the algorithm, we can design a  $gs$  threshold. If the average correlation between the core attributes of a granule that satisfies  $gs$  is above this threshold, it indicates that the correlation of these attributes is high and the probability that these attributes are on the same object is high, so we have reason not to perform  $gs$ .

Then for the  $gc$  operation, its core is to combine two granules that meet the conditions into one. Based on this, we can define a  $gc$  threshold. If the average correlation between the core attributes of the two granules is lower than this threshold, it means that the core attributes of the two granules are not correlated enough, that is, these attributes are unlikely to appear in the same object, so we will refuse to perform the  $gc$  operation.

Finally, for the  $gi$  operation, its core is to isomorphize a sub-granule of a granule to another granule that meets the conditions as a sub-granule. Therefore, we will take a similar operation to  $gc$  and design a  $gi$  threshold. If the average correlation between the core attribute of this sub-granule and the intent of another granule is lower than this threshold, it means that their correlation is not sufficient, so the  $gi$  operation will be rejected.

Fig. 5 shows an example. Here is a dataset containing attributes  $a, b, c, d, e$ , and  $f$ . We show a part of its correlation two-dimensional matrix and set the three default thresholds to 0.85, 0.15, and 0.15. The thresholds mentioned in subsequent experiments also take this default value unless otherwise stated. In the figure, there are some original granules that meet the GrC conditions. We query the correlation matrix to calculate the average value and compare it with the threshold. If the conditions are met, the corresponding GrC operation is executed and the new particle is accepted. Otherwise, it is not executed and the new granule is rejected. Each node in the figure represents an  $AG$ , and the letter in the node represents its core attribute  $AG.K$ . The intent  $AG.A$  of the node is the union of all the core attributes from the top  $AG$  to itself. The line segment with an arrow in the  $gs$  and  $gc$  examples represents

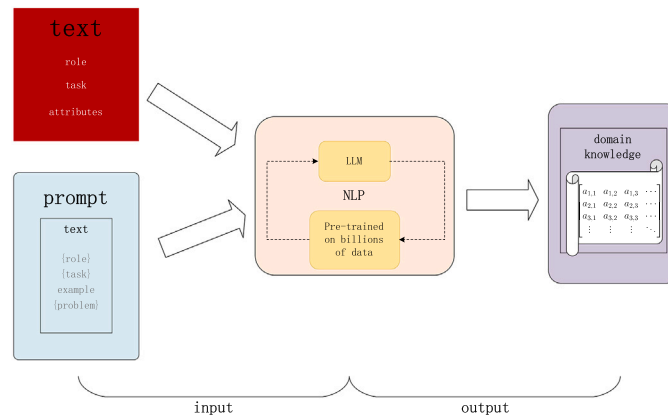


Fig. 4. Prompt for generating domain knowledge.

**Table 2**

Prompt template and an example for adjustment.

Role: {role}

Task: {task} If possible, give a probability value to indicate the likelihood of their co-occurrence, ranging from 0 (not at all possible) to 1 (completely possible).

Example:

- Attributes: Pleasant Goat, Beautiful Goat, Monocotyledon, Dicotyledon.

Attribute pair correlation matrix:

```

| Pleasant Goat|Beautiful Goat|Monocotyledon|Dicotyledon|
|-|-|-|-|
|Pleasant Goat|1.00|1.00|0.00|0.00|
|Beautiful Goat|1.00|1.00|0.00|0.00|
|Monocotyledon|0.00|0.00|1.00|0.00|
|Dicotyledon|0.00|0.00|0.00|1.00|

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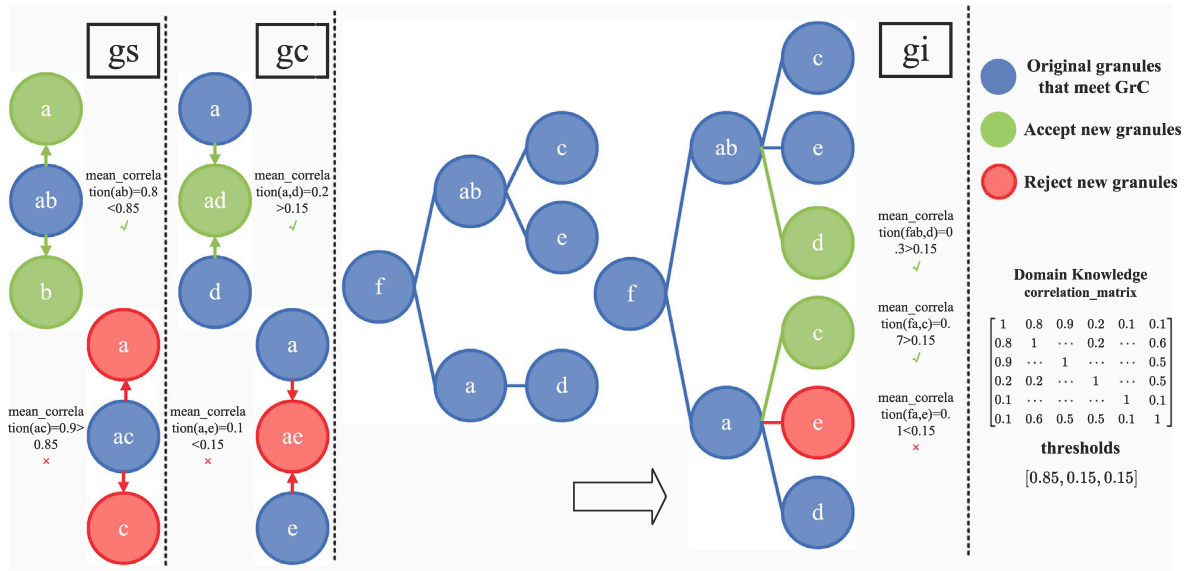
Now, please evaluate the following attributes and output the attribute pair correlation matrix.

- Attributes: {attributes}

role = "Biologist",

task = "Now you are discovering new species with the following attributes. Please evaluate whether the following attribute pairs are likely to appear in a species at the same time.",

attributes = "needs water to live, lives in water, lives on land, needs chlorophyll, dicotyledon, monocotyledon, can move, has limbs, breast feeds"

**Fig. 5.** An example of KR-POFSA with domain knowledge and thresholds.

GrC. For the first original AG, its core attribute  $AG.K = 'ab'$ , which meets the  $gs$  requirement and can be executed. At the same time, the correlation between **a** and **b** is 0.8, which is less than the  $gs$  threshold of 0.85, so the result of  $gs$  is accepted. For the fifth and sixth original AGs, their core attributes are 'a' and 'e' respectively. Since they also meet the  $gc$  requirements, new AGs can also be obtained, but because their correlation is 0.1, which is less than the  $gc$  threshold of 0.15, that is, the attributes are not related enough and are unlikely to appear in the same object, the result of  $gc$  is rejected. In the  $gi$  example, the line segment represents the partial order relationship of the attributes in the data set, with the upper layer on the left and the lower layer on the right. Consider the first  $gi$  result, its core attribute  $AG.K = 'd'$ , and its upper granule's intent  $AG_1.A = 'fab'$ , so their mean correlation is  $(0.5 + 0.2 + 0.2)/3 = 0.3$ , which is greater than the  $gi$  threshold of 0.15, so accept the result. The acceptance and rejection do not mean validity or invalidity, but indicate whether GrC is executed to obtain new AGs.

The thresholds for granule operations ( $gs = 0.85$ ,  $gc = 0.15$ ,  $gi = 0.15$ ) were selected through preliminary experiments evaluating multiple candidate values ( $gs : 1.00 \rightarrow 0.50$ ,  $step = -0.05$ ;  $gc/gi : 0.00 \rightarrow 0.50$ ,  $step = 0.05$ ). We prioritized the set that best matched our goals. By adjusting the values of these thresholds, we can effectively reduce the

### Algorithm 3.1 An algorithm for domain knowledge and KR-POFSA thresholds.

Input: *prompt\_template*: strOutput: *correlation\_matrix*: two-dimensional matrix(domain knowledge), *thresholds*: list(including  $gs$ ,  $gc$ ,  $gi$  threshold)

01. Design *role*, *task*, *attributes* according to the field of the dataset;
02. *prompt* = *prompt\_template*.format(*role*, *task*, *attributes*);
03. SelectLLMInput(*prompt*);
04. *correlation\_matrix* = ParseLLMOutput();
05. Adjust *correlation\_matrix*;
06. Design GrC threshold:  $gs$ ,  $gc$ ,  $gi$ , get *thresholds* = [*threshold\_splitting*, *threshold\_combination*, *threshold\_isomorphism*];
07. Return *correlation\_matrix*, *thresholds*;

number of new patterns that are finally discovered. It is worth mentioning that even if you keep reducing the results of discovery, the remaining results may not necessarily be useful.

**Algorithm 3.2** LLM-enhanced KR-POFSA (LLM-KR-POFSA).

---

Input:  $K = \{U, M, I\}$  and  $T = (V, E)$  (APOS-3W of  $K = \{U, M, I\}$ ),  
 $correlation\_matrix[][]$  (domain knowledge),  
 $thresholds[]$  (GrC threshold:  $gs, gc, gi$ )  
Output:  $P = \{(O, A)\}$  a collection of new potential  $OPs$

01.  $P = \{\}$ ,  $granuleLayer = 0$ ;
02.  $AGL = loadGranuleByLayer(T, granuleLayer)$ ;
03. While  $AGL \neq \emptyset$
04.    $AG = getGranule(AGL)$ ;
05.   While  $AG \neq \emptyset$
06.     If  $meanCorrelation(AG.K) \leq threshold\_splitting$
07.       doGranuleSplittingOperation( $AG$ );
08.     End of If block
09.      $AG = getGranule(AGL)$ ;
10.   End of While loop
11.    $AG_1, AG_2 = getGranule(AGL)$ ;
12.   While  $AG_1 \neq \emptyset$  and  $AG_2 \neq \emptyset$
13.     If  $meanCorrelation(AG_1.K, AG_2.K) \geq threshold\_combination$
14.       doGranuleCombinationOperation( $AG_1, AG_2$ );
15.     End of If block
16.      $AG_1, AG_2 = getGranule(AGL)$ ;
17.   End of While loop
18.    $AG_1, AG_2 = getGranule(AGL)$ ;
19.   While  $AG_1 \neq \emptyset$  and  $AG_2 \neq \emptyset$
20.     If  $meanCorrelation(AG_1.A, AG_2.K) \geq threshold\_isomorphism$
21.       doGranuleIsomorphismOperation( $AG_1, AG_2, P$ );
22.     End of If block
23.      $AG_1, AG_2 = getGranule(AGL)$ ;
24.   End of While loop
25.   If there are any new  $OP$  derived from  $gi$
26.     GOTO step11;
27.   End of If block
28.    $granuleLayer + 1$ ;
29.    $AGL = loadGranuleByLayer(T, granuleLayer)$ ;
30. End of While loop
31. Return  $P$ ;

---

**Algorithm 3.3** meanCorrelation.

---

Input:  $A1$  (set of attributes),  $A2$  (set of attributes, optional),  $CM = correlation\_matrix[][]$  (domain knowledge, optional)  
Output:  $mean\_corr$  (The mean value of all attribute correlations between  $A1$  and  $A2$ )

01.  $res = 0$
02. If  $A2 = NULL$
03.   For  $i$  from 0 to  $len(A1)-2$
04.     For  $j$  from  $i+1$  to  $len(A1)-1$
05.        $res += CM[A1[i] \cdot 'a'] [A1[j] \cdot 'a']$
06.      $mean\_corr = res / (len(A1) * (len(A1)-1) / 2)$
07. Else
08.   For  $i$  from 0 to  $len(A1)-1$
09.     For  $j$  from 0 to  $len(A2)-1$
10.        $res += CM[A1[i] \cdot 'a'] [A2[j] \cdot 'a']$
11.      $mean\_corr = res / (len(A1) * len(A2))$
12. End of If block
13. Return  $mean\_corr$ ;

---

**3.3. LLM-enhanced KR-POFSA**

According to these improvements, for a given APOS-3W, the few shot knowledge reasoning can be performed according to the following steps:

First of all, obtain domain knowledge based on datasets and prompt templates, and then design thresholds for KR-POFSA, which is specifically described in Algorithm 3.1.

After obtaining the domain knowledge and thresholds for KR-POFSA, we can perform knowledge reasoning based on the Algorithm 3.2 (LLM-KR-POFSA). The calculation process of mean correlation can be implemented with Algorithm 3.3.

Fig. 6 is a complete flowchart of our method. You will have a better understanding by looking at it.

**3.4. Analysis of LLM-KR-POFSA****3.4.1. Relationship between LLM output and KR-POFSA**

In both KR-POFSA and LLM-KR-POFSA, the partial order logic of the formal context  $K = \{U, M, I\}$  is implied in the APOS-3W constructed at the beginning. The process of using GrC ( $gs, gc, gi$ ) for KR is based on APOS-3W and does not destroy its basic structure, so the partial order logic of the formal context is not affected. For LLM-KR-POFSA, we use the correlation matrix between the attributes of the LLM output to constrain the GrC and reduce the number of new  $OPs$ , so it does not need to be consistent with the partial order logic, because this will not have any impact on it.

Although the domain knowledge (correlation matrix) output by the LLM does not need to be consistent with the partial order logic of the formal context, it needs to be compatible with the domain of the formal context. We achieve this requirement by adding role, task, and attributes to the prompt. And how do we ensure the accuracy and certainty of the output? This involves step 05 of Algorithm 3.1, adjusting the correlation matrix. LLM can output the correlation matrix we need, but it may be accompanied by some other unnecessary text information, which should be ignored directly. At the same time, we can also adjust the values in the correlation matrix according to our knowledge. For example, for two obviously mutually exclusive attributes ( $\exists a_i, a_j \in OP.A, s.t. a_i \perp a_j$ ), the correlation output by LLM is 0.05 or other values. Since we know that the two mutually exclusive attributes cannot appear in the same specific object, we can change the value to 0.00. More specific adjustments may require the intervention of experts in related fields. The use of LLMs for correlation matrix generation is motivated by practical scenarios where perfect prior knowledge is unavailable. When performing specific knowledge reasoning, it is obviously best if there are professional personnel to design the correlation matrix, GrC thresholds, and judge whether the new  $OP$  is valid.

**3.4.2. Time and space complexity**

For a given formal context  $K = \{U, M, I\}$  and its APOS-3W  $T = (V, E)$ , if we use method of brute force (BF) to reason knowledge, then the time complexity will be  $O(2^{|M|})$ , meanwhile, the number of patterns for attribute combinations will be extremely large, many of which are invalid and unreasonable.

As for KR-POFSA, its time complexity depends mainly on the knowledge reasoning operators:  $gs, gc, gi$ . Let  $LC$  represent the count of granular layers in the APOS-3W,  $KC$  represent the maximum count of core attributes among the attribute granules within the same layer of the APOS-3W,  $TC$  represent the maximum number of granules in the same layer of an APOS-3W structure that satisfy the demands of triadic closure theory,  $IC$  represent the maximum number of child granules in the same layer of an APOS-3W structure that satisfy the demands of granule isomorphism, thus the complexity of time is  $LC \times (2^{KC} + TC^2 + IC^2)$ .

Our improvements are mainly divided into two parts, one is to obtain domain knowledge and design thresholds for KR-POFSA, and the other is to perform LLM-enhanced KR-POFSA based on them. For the former, its steps are relatively simple. It only needs to input the prompt into the LLM and obtain the output, and then design the thresholds for KR-POFSA, so its time complexity can be regarded as  $O(1)$ . For the LLM-enhanced KR-POFSA, we mainly add a judgment condition before each execution of KR-POFSA. The time complexity of this statement is  $O(1)$ , so it has almost no effect on the final time complexity, that is, the time complexity is still  $LC \times (2^{KC} + TC^2 + IC^2)$ .

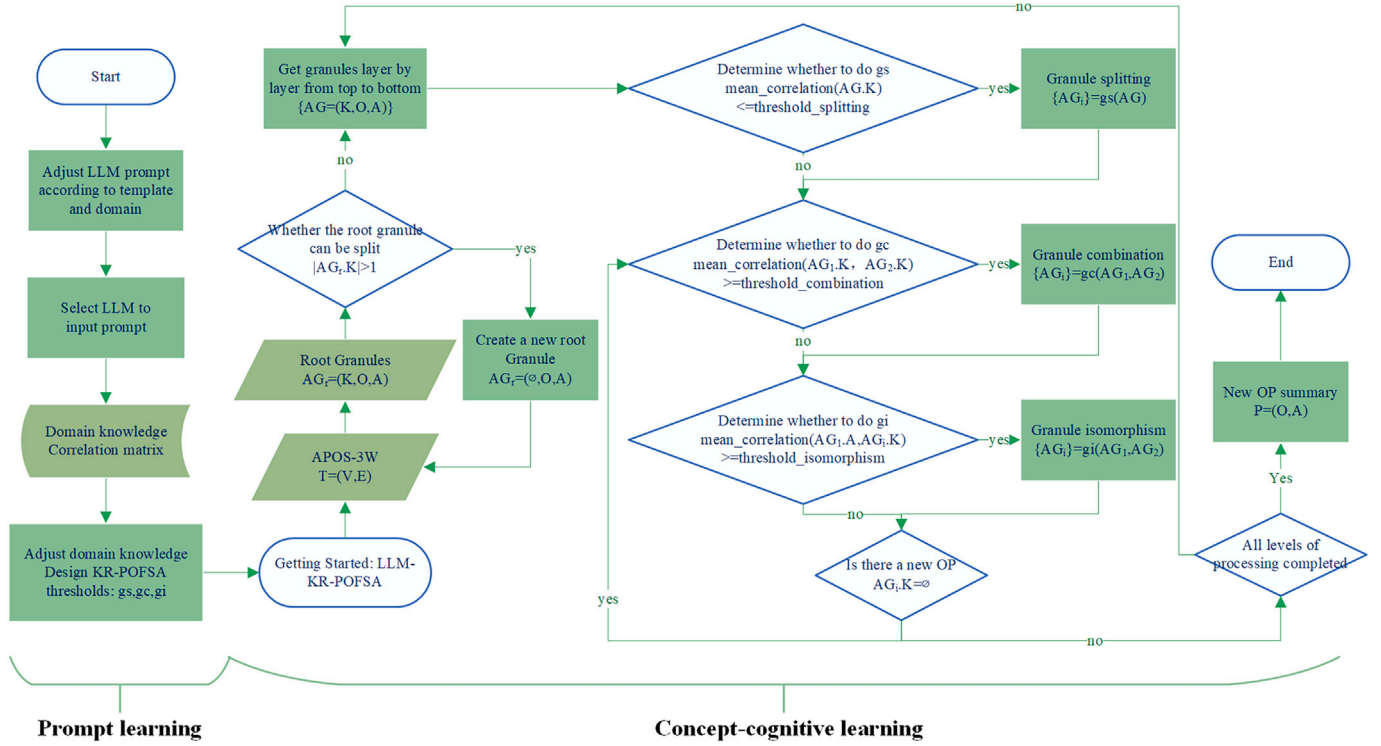


Fig. 6. The whole process of LLM-KR-POFSA.

All in all, in terms of time complexity, the order of magnitude is  $O(LLM-KR-POFSA) = O(KR-POFSA) < O(BF)$ . In terms of space complexity, except for the two-dimensional matrix of domain knowledge and the three thresholds for KR-POFSA, the rest of our method is the same as KR-POFSA, so our space complexity  $O(|M|^2)$  is larger than that of KR-POFSA.

## 4. Experiments

In this section we will further introduce and validate our method through experiments.

### 4.1. Datasets

We conducted experiments on three datasets with distinct domain characteristics:

The first is Live in water, a benchmark in FCA and CCL, which is shown in Table 1, containing 8 organisms characterized by 9 binary attributes describing survival environments. The attributes can be seen in Table 2. Each object-attribute pair indicates whether a species requires this condition. We use it to validate our method's capability in reconstructing known cognitive structures, with derived APOSSs visualized in Fig. 2.

The second is the dataset Math2015 Math1, it is a dataset about a final math examination: there are result datasets (scores on each problem) and problem datasets (skills required for each problem), we use the latter dataset to perform our method, and this dataset is shown in Table 3. It includes 20 mathematical problems (objects) mapped to 11 core competencies (attributes) in the Chinese secondary school curriculum standards, and the attributes can be seen in Table 5. It is used to test the accuracy of our method in skill decomposition in a real educational context.

The final dataset is on Tuberculosis (TB) from the Healthcare Informatics Project of the South African Medical Research Council. We also provide its data in Table 4. It contains 12 tuberculosis diagnostic

Table 3

Problem-skill datasets of Math1.

|    | a | b | c | d | e | f | g | h | i | j | k |
|----|---|---|---|---|---|---|---|---|---|---|---|
| 1  | 1 | 1 |   |   |   |   |   |   |   | 1 | 1 |
| 2  |   |   | 1 |   |   |   |   | 1 |   |   |   |
| 3  |   |   | 1 |   |   |   |   |   |   |   | 1 |
| 4  |   |   |   |   | 1 |   |   |   |   | 1 | 1 |
| 5  |   |   |   |   |   | 1 |   | 1 |   |   |   |
| 6  |   |   |   |   |   |   | 1 | 1 |   | 1 |   |
| 7  |   |   |   |   |   | 1 |   |   | 1 |   | 1 |
| 8  |   |   |   |   | 1 |   |   |   |   | 1 | 1 |
| 9  |   |   | 1 |   |   |   |   | 1 |   |   | 1 |
| 10 | 1 |   | 1 |   |   | 1 | 1 | 1 |   | 1 | 1 |
| 11 |   |   |   | 1 |   |   |   |   |   |   | 1 |
| 12 |   |   |   |   |   | 1 |   | 1 |   |   | 1 |
| 13 |   |   | 1 |   |   |   |   |   |   |   | 1 |
| 14 |   |   |   |   | 1 |   |   |   |   | 1 | 1 |
| 15 |   |   |   |   |   | 1 | 1 | 1 | 1 | 1 | 1 |
| 16 | 1 |   |   |   |   |   |   | 1 |   | 1 | 1 |
| 17 |   |   |   |   | 1 |   |   |   |   | 1 | 1 |
| 18 |   |   | 1 |   |   |   |   |   |   | 1 | 1 |
| 19 |   |   |   |   |   | 1 | 1 | 1 |   | 1 | 1 |
| 20 |   |   | 1 |   |   |   |   | 1 |   | 1 | 1 |

criteria (attributes) applied to 23 patient groups (objects), and these attributes can be seen in Table 6. This real-world medical dataset evaluates our method's ability to handle complex diagnostic patterns.

These datasets were selected to represent biological taxonomies, educational assessments, and clinical decision-making scenarios respectively, demonstrating our method's cross-domain applicability. All data matrices preserve original incidence relationships without preprocessing to maintain ecological validity.

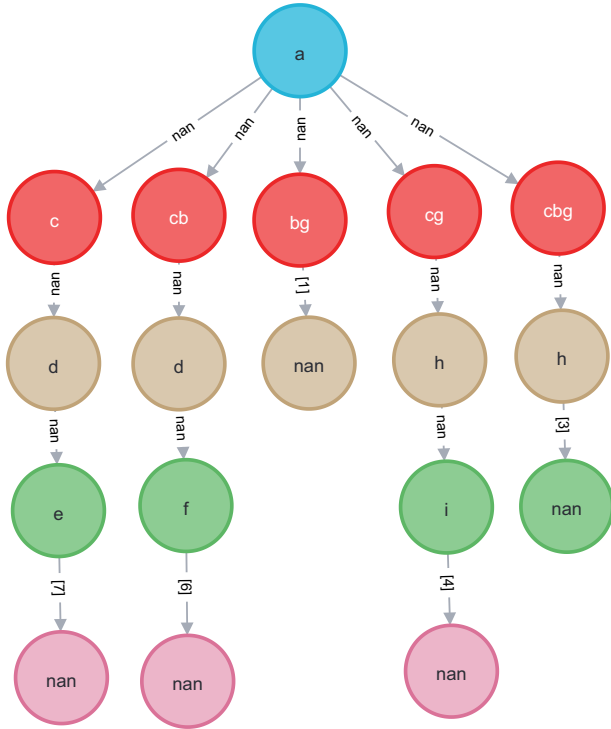
### 4.2. Processing

In order to verify the effectiveness of our LLM-enhanced KR-POFSA (LLM-KR-POFSA), we select some objects to form sub-datasets, and then



**Table 4**  
Tuberculosis.

|    | a | b | c | d | e | f | g | h | i | j | k | l |
|----|---|---|---|---|---|---|---|---|---|---|---|---|
| 1  | 1 | 1 | 1 |   |   | 1 |   |   | 1 | 1 |   |   |
| 2  | 1 | 1 | 1 |   |   | 1 | 1 | 1 | 1 |   |   |   |
| 3  | 1 | 1 | 1 |   |   | 1 | 1 | 1 | 1 |   | 1 | 1 |
| 4  |   |   |   |   | 1 |   |   |   |   | 1 | 1 |   |
| 5  |   |   |   |   |   | 1 |   | 1 |   |   |   |   |
| 6  |   |   |   |   |   |   | 1 | 1 |   | 1 |   |   |
| 7  |   |   |   |   |   | 1 |   |   | 1 |   | 1 |   |
| 8  |   |   |   |   | 1 |   |   |   |   | 1 | 1 |   |
| 9  |   |   | 1 |   |   |   |   | 1 |   |   | 1 |   |
| 10 | 1 |   | 1 |   |   | 1 | 1 | 1 |   | 1 | 1 |   |
| 11 |   |   |   | 1 |   |   |   |   |   |   | 1 |   |
| 12 |   |   |   |   |   | 1 |   | 1 |   |   | 1 |   |
| 13 |   |   | 1 |   |   |   |   |   |   |   | 1 |   |
| 14 |   |   |   |   | 1 |   |   |   |   | 1 | 1 |   |
| 15 |   |   |   |   |   | 1 | 1 | 1 | 1 | 1 | 1 |   |
| 16 | 1 |   |   |   |   |   |   | 1 |   | 1 | 1 |   |
| 17 |   |   |   |   | 1 |   |   |   |   | 1 | 1 |   |
| 18 |   |   | 1 |   |   |   |   |   |   | 1 | 1 |   |
| 19 |   |   |   |   |   | 1 | 1 | 1 |   | 1 | 1 |   |
| 20 |   |   | 1 |   |   |   |   | 1 |   | 1 | 1 |   |
| 21 |   |   | 1 |   |   |   |   | 1 |   | 1 | 1 |   |
| 22 |   |   | 1 |   |   |   |   | 1 |   | 1 | 1 |   |
| 23 |   |   | 1 |   |   |   |   | 1 |   | 1 | 1 |   |

**Fig. 7.** APOS-3W of sub Live in water.

perform our method on these sub-datasets. If the removed objects can be reasoned by LLM-KR-POFSA, we consider it effective.

For the first dataset Live in water, we select the 1st, 3rd, 4th, 6th, and 7th objects as sub-datasets to verify whether the 2nd, 5th, and 8th objects can be reasoned by KR-POFSA or our method. Fig. 7 is the APOS-3W we drew based on the sub Live in water.

For the second dataset Math2015 Math1, we select objects 2, 4, 8, 12, 14, 15, 16, and 20 as the objects to be reasoned, and the remaining 12 objects are used as sub-datasets in our experiment to reason about new objects. Fig. 8 shows the APOS-3W of the sub Math2015 Math1 problem-skill datasets.

As for the dataset TB, we select the last 9 objects as the objects to be reasoned about, and the remaining 14 objects as the sub-datasets of our experiment for reasoning about new objects. Fig. 9 shows the APOS-3W of the Tuberculosis dataset.

In Table 2, we provide the prompt template and an example, this example is the prompt we adjusted based on the sub Live in water datasets. The other two prompts we adjusted are shown in Table 5 and 6, respectively.

We use them to adjust the prompt template and input prompts into LLM (model=gpt-4o-2024-08-06, max\_tokens=1024, temperature=0.3, top\_p=0.95, seed=42) to get the output correlation matrix as shown in Table 7, 8, and 9, respectively.

After we obtain the output correlation matrix, we use 0.85, 0.15, 0.15 as default thresholds for  $gs$ ,  $gc$ , and  $gi$  to perform knowledge reasoning.

Based on these APOS-3W, prompts and domain knowledge, we can start knowledge reasoning with KR-POFSA and our LLM-enhanced KR-POFSA.

#### 4.3. Results

To begin with, we use KR-POFSA to reason about new objects, which is used for comparison with our method.

Fig. 10 shows the results of KR-POFSA and LLM-KR-POFSA on Live in water, respectively.

Due to the large number of AGs reasoned from the other two datasets, we will not present them here.

Table 10 presents the comparison of the results of KR-POFSA and LLM-enhanced KR-POFSA.

As mentioned in 3.2, we set the  $gs$  threshold from 1.00 to 0.50, and the  $gc$  and  $gi$  from 0.00 to 0.50 at intervals of 0.05 to obtain 11 groups of thresholds: (1.00, 0.00, 0.00), (0.95, 0.05, 0.05), ... (0.55, 0.45, 0.45), (0.50, 0.50, 0.50). We performed 11 LLM-enhanced KR-POFSA knowledge reasoning on the second and third datasets, and then get the proportion of the number of reasoned OP and AG relative to KR-POFSA, which is shown in Fig. 11. The figure also shows the proportion of test objects reasoned by the LLM-enhanced KR-POFSA relative to KR-POFSA as the threshold changes.

#### 4.4. Analysis

In this section, let us analyze the results together.

The first part is the analysis of the results on dataset Live in water. In the results of KR-POFSA and LLM-KR-POFSA, which are shown in Fig. 10, we marked the object patterns representing the 2nd, 5th, and 8th individuals. Their attributes are 'abgh', 'abdf', and 'acdf', respectively. We can see that both KR-POFSA and our LLM-KR-POFSA, which uses prompts to generate domain knowledge and then uses thresholds to restrict, can effectively reason about these three individuals, and as evidenced in Table 10, LLM-KR-POFSA reduces redundant AG/OP generation by 35.14 %/29.73 % compared to baseline, demonstrating our constraint mechanism's efficacy. As can be seen from Fig. 10, the reduction is mainly related to the combination of attribute granules of the two attributes 'e' and 'f', as well as the isomorphism of the two groups of granules 'ag' and 'd', and 'abg' and 'd'. Observe that the correlation between 'e' and 'f' in Table 7 is 0, which represents dicotyledon and monocotyledon, respectively. This is consistent with our common sense. It is impossible for an organism to be both a monocotyledon and a dicotyledon. For 'ag' and 'd', animals are generally able to move, while plants generally need chlorophyll, so it is reasonable not to perform isomorphism calculations on them. The same is true for 'abg' and 'd'.

Next, it is the analysis of the results on dataset Math2015 Math1. The attributes of the eight objects to be reasoned are 'cd', 'ejk', 'ejk', 'ejk', 'fhk', 'fghijk', 'ahjk', 'chjk'. Both KR-POFSA and our LLM-KR-POFSA can effectively reason about them. However, our method does not effectively reduce the number of new OPs reasoning. If you want to achieve this effect, you may find that reducing the number of reasoned OP and AG often leads to a reduction in the number of reasoned

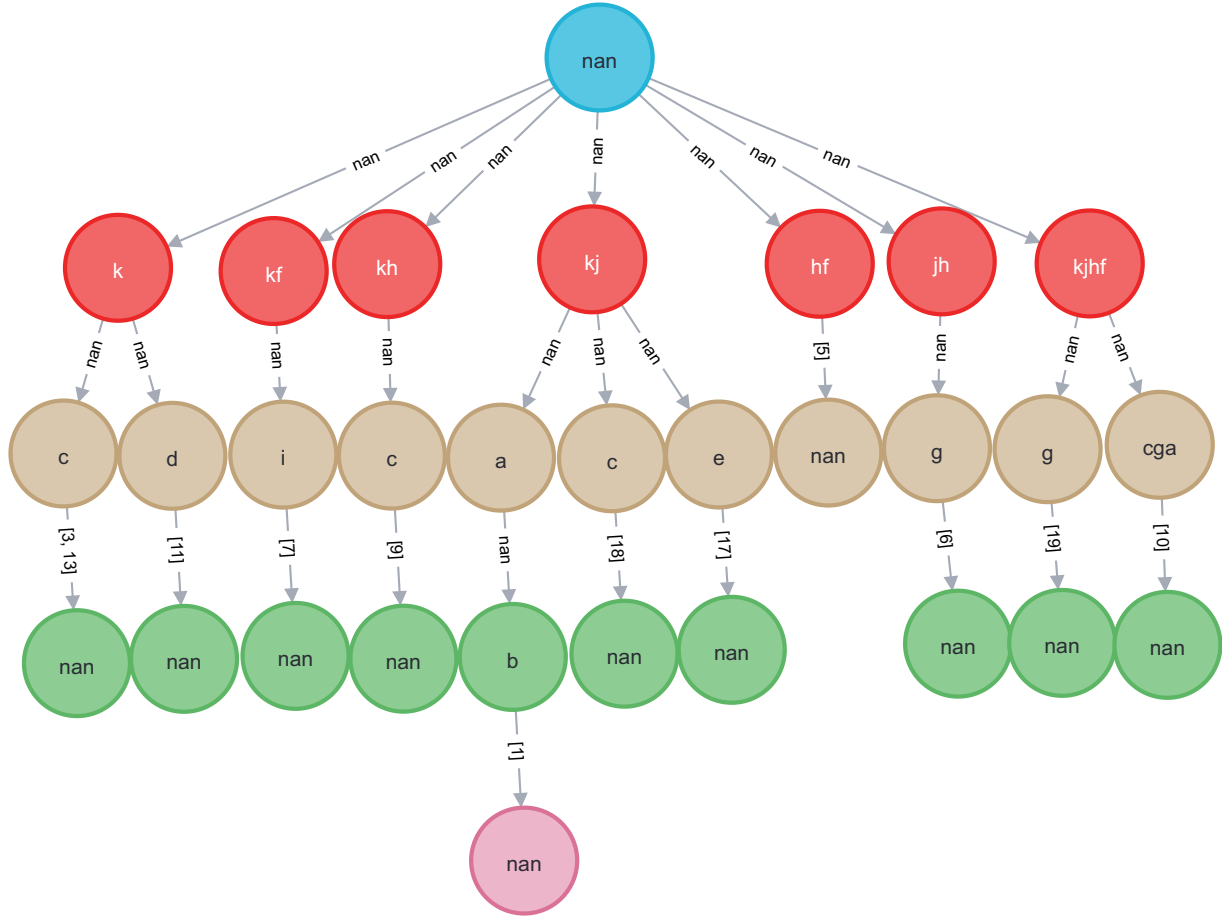


Fig. 8. APOS-3W of sub Math2015 Math1.

test objects as shown in Fig. 11. To maintain reasoning accuracy for critical patterns like test objects while suppressing irrelevant outputs, we propose three possible methods: prompt engineering refinement, correlation matrix calibration, and threshold adaptation with targeted attribute prioritization.

Finally, we analyze the results on dataset Tuberculosis. The object attributes that need to be reasoned are 'abcfghl', 'abcfghkl', 'abcfhijkl', 'abcfghkl', 'abcfghkl', 'abefghl', 'afgil', 'f', and 'fgh'. Under the default threshold, 'abefghl' can not be reasoned. We still conducted multiple experiments with 11 sets of thresholds to obtain Fig. 11(b), and we observe an inherent trade-off: aggressive pruning of Associated Granules (AG) and Object Properties (OP) inevitably degrades valid pattern discovery. And for this dataset, we see that if we want to reason all test objects, the number of useless patterns must be huge, but if we choose to perform knowledge reasoning under the default or smaller threshold, although we cannot reason the object 'abefghl', we can reduce the number of reasoned AG and OP to half.

As can be seen from Fig. 11, our method did not improve the performance on the two datasets in the initial 3, 2 groups of thresholds, but changes occurred in both datasets starting from the 4, 3 groups. For the second dataset, the threshold continued to change and caused several changes in the proportions. The number of new OPs found decreased, which also led to a decrease in the number of test OPs found. This is not acceptable. For the third dataset, there was basically no change in the subsequent period. Only at the end did it continue to decrease, which is clearly unacceptable. Therefore, we selected the third set of thresholds ( $gs = 0.85$ ,  $gc = 0.15$ ,  $gi = 0.15$ ) as the default thresholds. Why is the

performance degradation uneven across different datasets? This is due to the correlation of the attributes in the formal context. If the correlation of the attributes is high, gc and gi are mainly executed; if the correlation is low, gs is mainly executed; otherwise, all three operations may be executed.

All in all, if we want to reason valid patterns while reducing the generation of useless patterns, we can choose to further adjust the prompt, domain knowledge: correlation matrix, or use a smaller threshold but pay special attention to certain attributes. The tuberculosis dataset exemplifies the value of threshold adaptation. We choose a smaller threshold and then pay special attention to attribute e: clear sputum during knowledge reasoning. Although our common sense and experience tell us that the sputum of tuberculosis patients is not clear, we know from experiments that people with clear sputum may also have tuberculosis. This means that doctors need to pay attention to the symptom of clear sputum when diagnosing, because clear sputum may also indicate a tuberculosis patient, to avoid misdiagnosis and missed diagnosis.

## 5. Discussion

### 5.1. Discussion of value

Knowledge makes intelligence, the essence of intelligence is how to realize knowledge reasoning, while the key to knowledge reasoning lies in the appropriate method of knowledge representation. Therefore, knowledge representation and reasoning are the foundations of artificial intelligence (AI) [51]. Over the past few decades, various knowledge reasoning methods have been proposed to address different reasoning tasks,

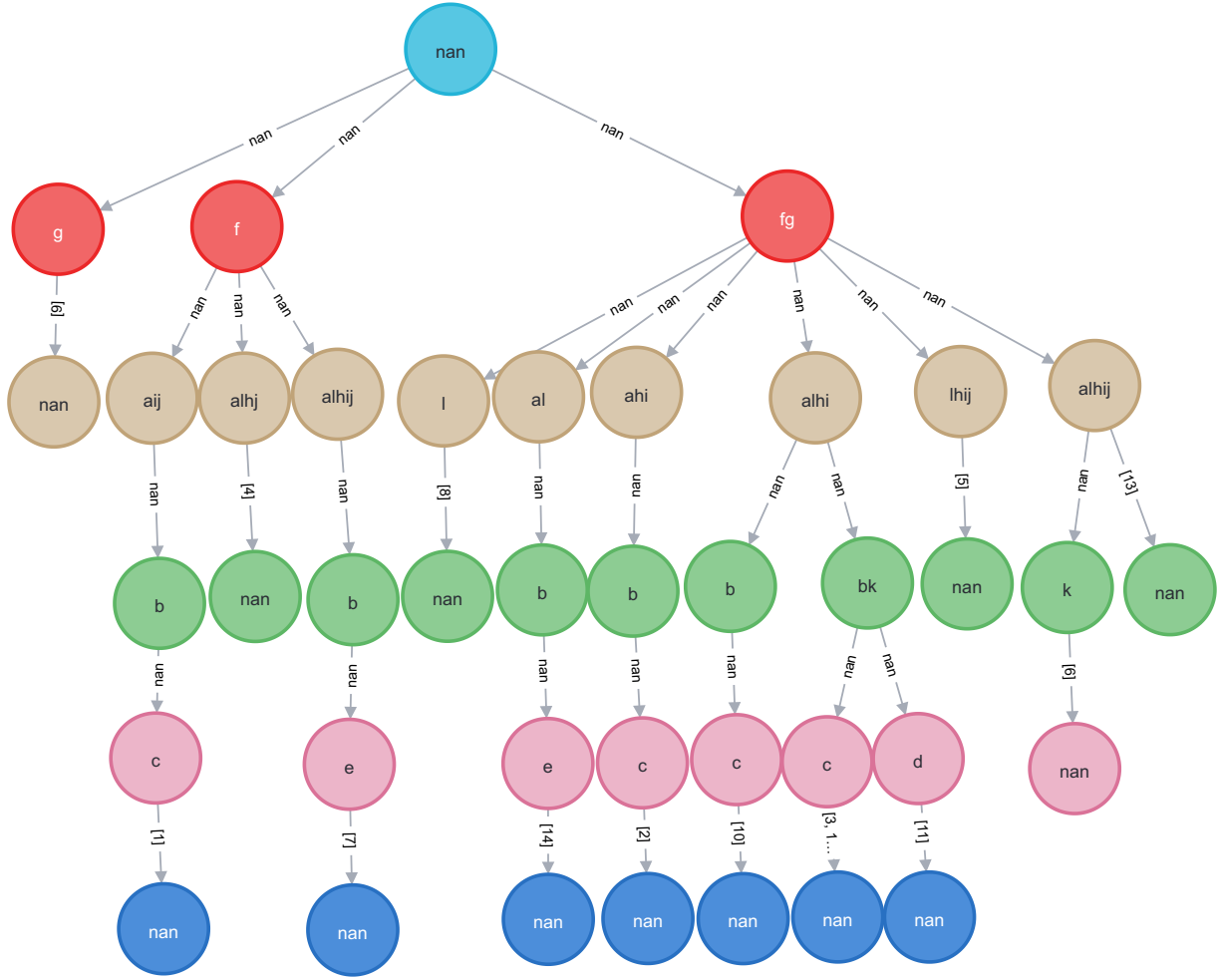


Fig. 9. APOS-3W of sub Tuberculosis.

Table 5

Adjusted prompt based on sub Math2015 Math1.

|                                                                                                                                                                                                                                            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>role</b> = "Mathematics teacher",                                                                                                                                                                                                       |
| <b>task</b> = "You are now writing questions for an exam. There are the following test points. Please evaluate whether the following test points are likely to appear in the same question at the same time.",                             |
| <b>attributes</b> = "Set, Inequality, Trigonometric function, Logarithm versus exponential, Plane vector, Property of function, Image of function, Spatial imagination, Abstract summarization, Reasoning and demonstration, Calculation." |

Table 6

Adjusted prompt based on sub Tuberculosis.

|                                                                                                                                                                                                                                            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>role</b> = "Respiratory medicine, infectious disease doctor",                                                                                                                                                                           |
| <b>task</b> = "Now you are finding a tuberculosis patient, and now have the following symptoms, please evaluate whether the following symptoms are likely to appear in the same tuberculosis patient at the same time.",                   |
| <b>attributes</b> = "Persistent cough, Sputum production, Sputum produced is muco-purulent, Bloody sputum, Clear sputum, Weight loss, Extreme night sweats, No appetite, Chest pain, Shortness of breath, Tuberculosis contact, Tiredness" |

which can be roughly divided into two categories from the perspective of knowledge representation: symbol-based logical reasoning [46] and vector-based representation learning [21].

At present, knowledge graphs (KGs) and pre-trained language models (PLMs) can be regarded as the representative knowledge representation methods for symbol-based logical reasoning and

vector-based representation learning, respectively. KGs and PLMs are both technical means of representing and processing knowledge, and can accomplish different reasoning tasks based on knowledge.

For PLMs, their reasoning mainly relies on neural networks to operate in parameterized vector space, so it is relatively implicit. The reasoning supported by PLMs has excellent generalization ability, but

**Table 7**  
Correlation matrix of sub Live in Water.

|   | a   | b   | c   | d   | e   | f   | g   | h   | i   |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| a | 1.0 | 0.8 | 0.5 | 0.2 | 0.2 | 0.2 | 0.5 | 0.3 | 0.4 |
| b | 0.8 | 1.0 | 0.0 | 0.1 | 0.0 | 0.0 | 0.3 | 0.1 | 0.2 |
| c | 0.5 | 0.0 | 1.0 | 0.3 | 0.4 | 0.3 | 0.7 | 0.6 | 0.7 |
| d | 0.2 | 0.1 | 0.3 | 1.0 | 0.8 | 0.8 | 0.0 | 0.0 | 0.0 |
| e | 0.2 | 0.0 | 0.4 | 0.8 | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| f | 0.2 | 0.0 | 0.3 | 0.8 | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| g | 0.5 | 0.3 | 0.7 | 0.0 | 0.0 | 0.0 | 1.0 | 0.9 | 0.8 |
| h | 0.3 | 0.1 | 0.6 | 0.0 | 0.0 | 0.0 | 0.9 | 1.0 | 0.9 |
| i | 0.4 | 0.2 | 0.7 | 0.0 | 0.0 | 0.0 | 0.8 | 0.9 | 1.0 |

**Table 8**  
Correlation matrix of sub Math2015 Math1.

|   | a   | b   | c   | d   | e   | f   | g   | h   | i   | j   | k   |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| a | 1.0 | 0.8 | 0.6 | 0.6 | 0.5 | 0.7 | 0.6 | 0.4 | 0.5 | 0.8 | 0.9 |
| b | 0.8 | 1.0 | 0.5 | 0.7 | 0.3 | 0.6 | 0.5 | 0.2 | 0.4 | 0.7 | 0.8 |
| c | 0.6 | 0.5 | 1.0 | 0.6 | 0.2 | 0.5 | 0.7 | 0.1 | 0.3 | 0.5 | 0.8 |
| d | 0.6 | 0.7 | 0.6 | 1.0 | 0.2 | 0.5 | 0.6 | 0.1 | 0.3 | 0.5 | 0.8 |
| e | 0.5 | 0.3 | 0.2 | 0.2 | 1.0 | 0.3 | 0.2 | 0.7 | 0.2 | 0.4 | 0.6 |
| f | 0.7 | 0.6 | 0.5 | 0.5 | 0.3 | 1.0 | 0.5 | 0.3 | 0.5 | 0.6 | 0.8 |
| g | 0.6 | 0.5 | 0.7 | 0.6 | 0.2 | 0.5 | 1.0 | 0.1 | 0.3 | 0.5 | 0.8 |
| h | 0.4 | 0.2 | 0.1 | 0.1 | 0.7 | 0.3 | 0.1 | 1.0 | 0.2 | 0.3 | 0.4 |
| i | 0.5 | 0.4 | 0.3 | 0.3 | 0.2 | 0.5 | 0.3 | 0.2 | 1.0 | 0.5 | 0.7 |
| j | 0.8 | 0.7 | 0.5 | 0.5 | 0.4 | 0.6 | 0.5 | 0.3 | 0.5 | 1.0 | 0.8 |
| k | 0.9 | 0.8 | 0.8 | 0.8 | 0.6 | 0.8 | 0.8 | 0.4 | 0.7 | 0.8 | 1.0 |

**Table 9**  
Correlation matrix of sub Tuberculosis.

|   | a   | b   | c    | d    | e    | f    | g    | h    | i    | j    | k   | l    |
|---|-----|-----|------|------|------|------|------|------|------|------|-----|------|
| a | 1.0 | 0.9 | 0.7  | 0.6  | 0.1  | 0.8  | 0.7  | 0.6  | 0.5  | 0.5  | 0.9 | 0.8  |
| b | 0.9 | 1.0 | 0.7  | 0.6  | 0.1  | 0.8  | 0.7  | 0.6  | 0.5  | 0.5  | 0.9 | 0.8  |
| c | 0.7 | 0.7 | 1.0  | 0.5  | 0.05 | 0.6  | 0.5  | 0.5  | 0.4  | 0.4  | 0.7 | 0.6  |
| d | 0.6 | 0.6 | 0.5  | 1.0  | 0.05 | 0.6  | 0.5  | 0.5  | 0.4  | 0.4  | 0.7 | 0.6  |
| e | 0.1 | 0.1 | 0.05 | 0.05 | 1.0  | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.1 | 0.05 |
| f | 0.8 | 0.8 | 0.6  | 0.6  | 0.05 | 1.0  | 0.7  | 0.7  | 0.6  | 0.6  | 0.9 | 0.8  |
| g | 0.7 | 0.7 | 0.5  | 0.5  | 0.05 | 0.7  | 1.0  | 0.6  | 0.5  | 0.5  | 0.8 | 0.7  |
| h | 0.6 | 0.6 | 0.5  | 0.5  | 0.05 | 0.7  | 0.6  | 1.0  | 0.5  | 0.5  | 0.8 | 0.7  |
| i | 0.5 | 0.5 | 0.4  | 0.4  | 0.05 | 0.6  | 0.5  | 0.5  | 1.0  | 0.7  | 0.7 | 0.6  |
| j | 0.5 | 0.5 | 0.4  | 0.4  | 0.05 | 0.6  | 0.5  | 0.5  | 0.7  | 1.0  | 0.7 | 0.6  |
| k | 0.9 | 0.9 | 0.7  | 0.7  | 0.1  | 0.9  | 0.8  | 0.8  | 0.7  | 0.7  | 1.0 | 0.9  |
| l | 0.8 | 0.8 | 0.6  | 0.6  | 0.05 | 0.8  | 0.7  | 0.7  | 0.6  | 0.6  | 0.9 | 1.0  |

lacks interpretability. In addition, due to the dependence of the reasoning process on the generation process, it is easy to generate illusions, resulting in a lack of reliability in the reasoning results [28].

For KGs, their reasoning mainly relies on explicitly obtained symbolic knowledge to complete, so the interpretability is relatively good and the reliability of the reasoning process is high. With the application of graph neural networks and other knowledge representation methods to knowledge graphs, it is also possible to do reasoning on KGs in vector space. However, overall, due to the limited knowledge coverage of KGs compared to PLMs, their reasoning process relies heavily on symbolic computation, resulting in poor generalization ability [22].

In view of the respective advantages and disadvantages of reasoning supported by KGs and PLMs, scholars have begun to seek methods that can combine both symbol-based logical reasoning and vector-based representation learning, which is called neuro-symbolic AI [6,10,11,26,47].

We consider that this article can be seen as an attempt at neuro-symbolic AI in theory. Concepts, as the cornerstone of human cognition, are also an important component of the KGs. KR-POFSA, as a concept-centered knowledge reasoning model, belongs to the category of symbol-based logical reasoning; while LLM, as a typical representative

of PLMs, belongs to the category of vector-based representation learning. Therefore, the LLM-KR-POFSA proposed in this article can be seen as a neuro-symbolic AI method to some extent.

## 5.2. Discussion of necessity

In the contemporary era, data, information, knowledge, understanding, and wisdom are increasingly recognized as the new oil that drives economic growth and innovation [14]. This notion is further underscored by O'Reilly Media's assertion that "the future belongs to the companies and people that turn data into products." As AI technologies accelerate, the significance of knowledge representation and reasoning becomes paramount, serving as the cornerstone for enhancing AI systems.

FSL emerges as a critical approach in scenarios where data acquisition is challenging. FSL's ability to learn from small datasets addresses a significant concern in fields where extensive data collection is impractical. Concurrently, Partially Ordered Formal Concept Analysis (POFSA) offers a robust mathematical framework, characterized by intuitiveness, formalization, and interpretability, making it a valuable tool for concept analysis and formal background visualization. APOS-3W, a knowledge representation model of POFSA, further enhances this by expressing associations between objects and hierarchical relationships between attributes. The integration of FSL and APOS-3W is thus imperative, as it leverages the strengths of both methodologies to address complex reasoning tasks. This combination not only enhances the interpretability of AI models but also broadens their applicability in diverse domains.

Domain knowledge, traditionally sourced from experts, is a cornerstone of AI's reasoning capabilities. LLMs, pre-trained on billions of data, contain rich domain knowledge. Prompting, as a technology to guide LLM output, can help large models output the domain knowledge they contain. Consequently, the design of prompt templates is crucial for harnessing LLMs' potential in knowledge representation and reasoning. In real life, when we reason, we often consider existing knowledge. However, various types of knowledge are complex and we cannot understand all knowledge. For example, though you know biological species, but you may not know math problems or tuberculosis. This underscores the significance of guiding LLMs with prompts to output domain-specific knowledge, thereby enhancing their reasoning capabilities.

Our proposed LLM-enhanced KR-POFSA approach integrates these elements, offering a powerful framework for knowledge reasoning. By combining the strengths of LLMs, FSL, and APOS-3W, this approach addresses the limitations of traditional methods, providing a more comprehensive and flexible solution.

In conclusion, the necessity of integrating LLMs with APOS-3W and FSL lies in their collective ability to enhance knowledge representation and reasoning, thereby advancing AI's potential in various applications. This integration not only aligns with contemporary AI research trends but also addresses practical challenges in data scarcity and knowledge complexity.

## 5.3. Discussion of limitations

In our work, the threshold value is not rigorous enough. We designed 11 groups of thresholds for experiments and selected them based on the number of test *OPs* and reasoning *OPs*. In fact, they should be selected based on the number of valid *OPs* and reasoning *OPs*. However, the judgment of valid *OPs* requires the support of external databases or domain experts, which requires a lot of manpower and time. Another method is to systematically optimize the threshold according to certain rules. There is no doubt that it also requires time for further thinking and research.



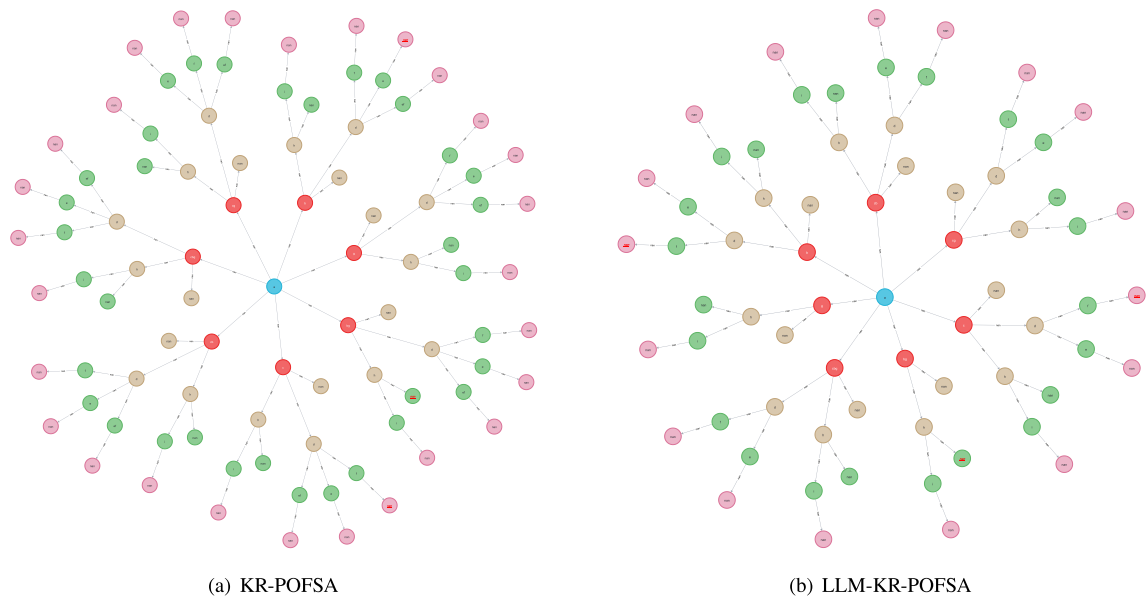


Fig. 10. The results of knowledge reasoning for Live in water.

Table 10  
Comparison of KR-POFSA and LLM-KR-POFSA results.

| Dataset       | Layer | Attribute Count | Granule Count |          |              | Pattern Count |          |              |
|---------------|-------|-----------------|---------------|----------|--------------|---------------|----------|--------------|
|               |       |                 | Initial       | KR-POFSA | LLM-KR-POFSA | Initial       | KR-POFSA | LLM-KR-POFSA |
| Live in water | L1    | 1               | 1             | 0        | 0            | 0             | 1        | 0            |
|               | L2    | 3               | 5             | 2        | 2            | 1             | 6        | 6            |
|               | L3    | 2               | 4             | 10       | 8            | 1             | 6        | 6            |
|               | L4    | 3               | 3             | 25       | 14           | 3             | 25       | 14           |
|               | Sum   | 9               | 13            | 37       | 24           | 5             | 37       | 26           |
| Math2015      | L1    | 0               | 1             | 0        | 0            | 0             | 0        | 0            |
| Math1         | L2    | 4               | 7             | 8        | 8            | 1             | 14       | 14           |
|               | L3    | 6               | 10            | 339      | 332          | 340           | 6        | 333          |
|               | L4    | 1               | 1             | 348      | 341          | 1             | 348      | 341          |
|               | Sum   | 11              | 19            | 695      | 681          | 11            | 702      | 688          |
| Tuberculosis  | L1    | 0               | 1             | 0        | 0            | 0             | 0        | 0            |
|               | L2    | 2               | 3             | 0        | 0            | 1             | 2        | 2            |
|               | L3    | 5               | 9             | 84       | 84           | 4             | 89       | 89           |
|               | L4    | 2               | 7             | 272      | 272          | 1             | 278      | 278          |
|               | L5    | 3               | 7             | 1946     | 832          | 7             | 1946     | 832          |
|               | Sum   | 12              | 27            | 2302     | 1188         | 13            | 2315     | 1201         |

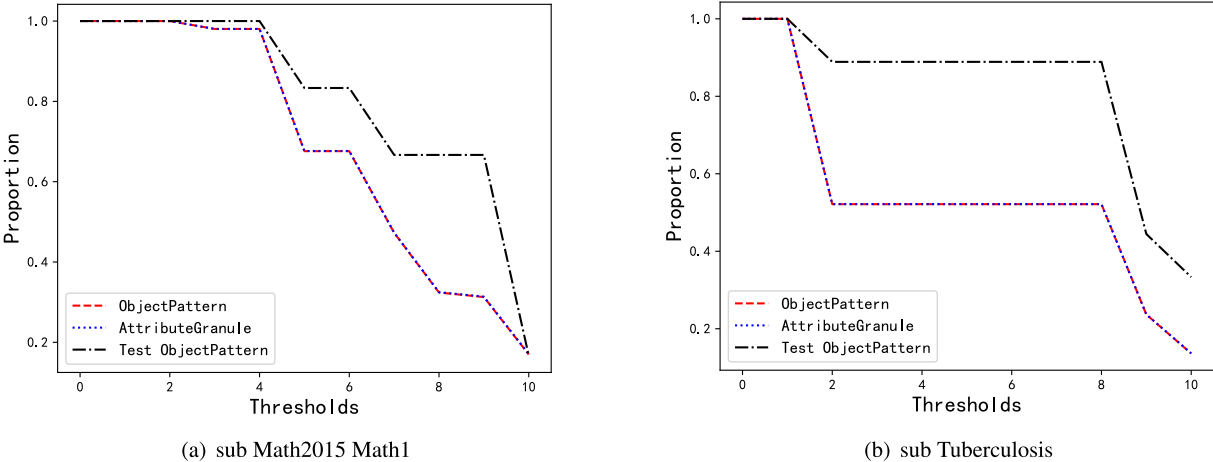


Fig. 11. The proportion of OP, AG, and test individuals generated.

Additionally, we found that although under certain threshold constraints, the number of *AG* and *OP* reasoning is indeed reduced, it often leads to a reduction in valid pattern reasoning. In the Math2015 Math1 and Tuberculosis experiments, we have proposed some solutions. Here we give another one, which is to collect datasets and train or fine-tune a large model that specifically outputs attribute correlations [20]. This may obtain more accurate domain knowledge and reduce the consumption of manpower and time costs.

Additionally, we lack interaction. There is the WisTech meta-equation:

$$WISDOM = INTERACTIONS + ADAPTIVE JUDGEMENT + KNOWLEDGE \quad (5.1)$$

In KR-POFSA, the APOS-3W generated by the formal context is used as knowledge, and then in the LLM-KR-POFSA that we proposed, we use domain knowledge and knowledge reasoning thresholds as adaptive judgment. In other words, the introduction of interactions is a possible direction. Therefore, in the future, we can add more interactions to our method, and extend the existing LLM-enhanced KR-POFSA method to Interactive LLM-KR-POFSA.

## 6. Conclusion

This article mainly explores the few-shot knowledge reasoning mechanism from the perspective of adaptive judgment between LLM and KR-POFSA. In this article, we use several experiments to demonstrate our proposed method not only reduces the discovery of invalid object patterns in KR-POFSA by half, but also may offer practitioners insights into which attributes to prioritize, minimizing empiricism. This work constitutes an advancement in neuro-symbolic AI, adeptly merging the strengths of symbol-based logical reasoning (KR-POFSA) with vector-based representation learning (LLMs). Moving forward, potential research avenues include refining prompts to enhance LLM domain knowledge output, designing more precise thresholds, developing specialized models through fine-tuning, and investigating interactive methods to bolster knowledge reasoning capabilities.

## CRedit authorship contribution statement

**Yuxuan Huang:** Writing – original draft, Software. **Enliang Yan:** Writing – review & editing, Writing – original draft, Funding acquisition, Conceptualization. **Peiming Zhang:** Writing – review & editing, Funding acquisition. **Tianyong Hao:** Writing – review & editing, Funding acquisition.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix A. Acronyms and symbols

See Table 11.

**Table 11**

Acronyms and Symbols employed in this article.

| Acronyms                                    | Symbols |
|---------------------------------------------|---------|
| Knowledge reasoning                         | KR      |
| Few-shot learning                           | FSL     |
| Pre-trained language model                  | PLM     |
| Large language model                        | LLM     |
| Formal concept analysis                     | FCA     |
| Concept-cognitive learning                  | CCL     |
| Partial order formal structure analysis     | POFSA   |
| Attribute partial order structure           | APOS    |
| Three-way attribute partial order structure | APOS-3W |
| three-way decision                          | 3WD     |
| granular computing                          | GrC     |
| attribute granule                           | AG      |
| object pattern                              | OP      |
| Granule splitting                           | gs      |
| Granule combination                         | gc      |
| Granule isomorphism                         | gi      |

## Appendix B. Datasets

- (1) ‘Live in Water’
- (2) ‘Math 2015’
- (3) ‘Tuberculosis [17]’

## Data availability

Data will be made available upon request.

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